Abstracts

Some results of Luna in positive characteristic

Michael Bate
(University of York)

Luna's celebrated Étale Slice Theorem for actions of reductive groups on affine varieties has many consequences and applications in characteristic zero. There are, however, some fundamental problems with getting an analogue of the Slice Theorem in positive characteristic (non-separability of orbit maps, non-semisimplicity of representations for reductive groups). The aim of this talk is to show that, despite this, one can get at some of the results following from the Slice Theorem using other means.

Counting resolutions of symplectic quotient singularities

Gwyn Bellamy
(University of Glasgow)

If V is a symplectic vector space and G a finite subgroup of Sp(V), then the quotient singularity V/G is a very interesting object to study, both from the geometric and representation-theoretic point of view. One of the motivational problems in trying to understand the singularities of V/G is that of deciding whether V/G admits a symplectic resolution or not. More generally, one can ask how many symplectic resolutions it admits. The goal of this talk is to explain how one can count the number of symplectic resolutions of V/G. We'll present an explicit formula for this number in terms of the dimension of a certain Orlik-Solomon algebra. The key to deriving this formula is to relate the resolutions of V/G to the Calogero-Moser deformations, where one can use the representation theory of symplectic reflection algebras.

Parabolic conjugation on varieties of nilpotent matrices

Magdalena Boos
(Bergische Universität Wuppertal)

Let's have a look at the very general question 'What can we find out about parabolic conjugation on varieties of nilpotent matrices?' and concider it's various possible directions during the talk. One possible approach to find answers is given by a translation to the representation theory of a specific quiver, which we will discuss first. Concerning results, we start by discussing those actions admitting only a finite number of conjugacy classes and give an overview of interesting facts and questions still to be answered.

Factoriality for the reductive Zassenhaus variety and quantum enveloping algebra

Amiram Braun
(University of Haifa, Israel)

Let U denote either (1): the enveloping algebra of a reductive finite dimensional Lie algebra over an algebraically closed field of prime characteristic, or (2): the simply connected quantum enveloping algebra at a root of unity of a complex semi-simple finite dimensional Lie algebra.

We show, by similar proofs, that the center of U, in both cases, is factorial. While the first result was established by Tange (by different methods), the second one confirms a conjecture of Braun-Hajarnavis.

In our talk, we shall present (time permiting) the context and the methods involved in the proofs.

Lusztig's correspondence for spherical conjugacy classes

Giovanna Carnovale
(University of Padova)

G. Lusztig has recently introduced a bijection between certain unions of conjugacy classes in a reductive algebraic group G, called strata, and certain unions of conjugacy classes in the Weyl group W of G. I will show that strata are unions of sheets in G and that Lusztig's bijection restricts to a correspondence between strata of spherical conjugacy classes in G and (certain unions of) classes of involutions in W.

Composition factors of a tensor product of truncated symmetric powers

Stephen Donkin
(University of York)

This is joint work with H. Geranios. We are interested in the set of composition factors of an m-fold tensor product $S(E)^{\otimes m}$ of the symmetric algebra of the natural module E for a general linear group $\mathrm{GL}_n(K)$ over an algebraically closed field of positive characteristic p.

One also has the truncated symmetric power $\bar{S}(E)$ of E (obtained by factoring out the ideal generated by pth powers of positive degree elements). We relate the above problem to that of finding the set of composition factors of $\bar{S}(E)^{\otimes m}$, and this we are able to solve in complete generality. We use this to solve the original problem in case $m = 2$ and this leads to a generalisation of a theorem of Krop and Sullivan describing the composition factors of $S(E)$ as a twisted tensor product.

Schur-Weyl duality over finite fields

Stephen Doty
(Loyola University Chicago)

Schur-Weyl duality is a double-centralizer property for the natural commuting actions of the general linear group $GL_n(k)$ and the symmetric group Σ_r on the vector space $(k^n)^{\otimes r}$, with Σ_r acting by "place-permutation". Although originally proved in 1927 over the field $k = \mathbb{C}$ of complex numbers by Issai Schur, the same result actually holds for any infinite field k, and - when properly reformulated - there is a version for commutative rings (R.M. Thrall, J.A. Green, DeConcini-Procesi, Carter-Lusztig, etc.). This talk will focus of what happens over finite fields. It turns out that the classical result is still true, provided only that the field k has "enough" elements. A precise lower bound is available. This result is joint work with Dave Benson [Schur–Weyl duality over finite fields, Archiv der Mathematik (Basel) 93 (2009), 425-435]. I will try to give some details about the proof and make some related remarks.

On Borel Schur algebras

Karin Erdmann
(University of Oxford)

Borel Schur algebras $S^+(n, r)$ are the Schur algebras of degree r, corresponding to the group B^+ of upper triangular matrices in $GL_n(\mathbb{K})$ where $\mathbb{K}$ is an infinite field.
The first part of the lecture is a survey of basic properties and an application. Then we describe recent results on Auslander-Reiten sequences ending in simple modules. Furthermore we classify Borel Schur algebras of finite representation type.

[Joint work with A.P. Santana and I. Yudin.]

Graded Cartan invariants of symmetric groups

Anton Evseev
(University of Birmingham)

We consider a certain matrix with entries in Laurent polynomial ring $\mathbb{Z}[v, v^{-1}]$. It may be defined either as the Gram matrix of the Shapovalov form on (a part of) the basic representation of an affine quantum group of type A or as the graded Cartan matrix of a block of a symmetric group (or a Hecke algebra). When one specialises to $v = 1$, it becomes a generalized Cartan matrix studied by Külshammer, Olsson, and Robinson. I will state a conjecture asserting that the original matrix can be put into a diagonal form by row and column operations over $\mathbb{Z}[v, v^{-1}]$ and describing the resulting diagonal entries. I will also present evidence for the conjecture: a number of its consequences, obtained by specialising the ring $\mathbb{Z}[v, v^{-1}]$ (e.g. by replacing v with an integer) have recently been proved. This is joint work with Shunsuke Tsuchioka.

From super Harish-Chandra pairs to affine supergroups

Fabio Gavarini
(Università di Roma)

I present a new functorial construction that, with each super Harish-Chandra pair, associates an affine supergroup: this functor is then proved to be a quasi-inverse to the natural functor from affine supergroups (up to details) to super Harish-Chandra pairs, so the two functors actually yield equivalences between the category of super Harish-Chandra pairs and that of affine supergroup. The existence of similar equivalences was known (possibly in different contexts, such as the smooth or the complex analytic one), but the construction I present is actually new — I provide a different quasi-inverse functor — and extends to a much larger setup, with a totally different, more geometrical method.

Reference: arXiv:1308.0462, to appear in "Transactions of the AMS"

Finite-dimensional representations of maximal compact subalgebras of split real Kac-Moody algebras

Ralf Koehl
(JLU Giessen)

Damour et. al and Henneaux et al. "observed" spin representations for the maximal compact subalgebra of the split real Kac-Moody algebra of type E(10) in string theory.

As it turns out, such spin representations exist for arbitrary symmetrizable generalized Cartan matrices. If one restricts one's attention to the E(n) series one obtains a Cartan-Bott periodicity that stems from the theory of Clifford algebras. In case of E(6), E(7), E(8) this spin representation in injective, thus this Cartan-Bott periodicity provides an alternative elementary proof for the structure of the maximal compact subalgebras for these finite-dimensional Lie algebras.

These spin representations can also be integrated to group level, thus providing finite-dimensional representations of certain double "spin" covers of the maximal compact subgroups of the corresponding Kac-Moody groups.

The multiplicative eigenvalue problem and deformed quantum cohomology

Shrawan Kumar
(University of North Carolina, Chapel Hill)

This is a joint work with Prakash Belkale. Let G be a simple, connected, simply-connected complex algebraic group. Choose a Borel subgroup B and a maximal torus $H \subset B$. We denote the Lie algebras of G, B, H by the corresponding Gothic characters: $\mathfrak{g}, \mathfrak{b}, \mathfrak{h}$ respectively. Let $R \subset \mathfrak{h}^*$ be the set of roots of $\mathfrak{g}$ and let R^+ be the set of positive roots (i.e., the set of roots of $\mathfrak{b}$). We denote by Δ the set of simple roots $\{\alpha_1, \dots, \alpha_\ell\}$.

Consider the *fundamental alcove* $\mathscr{A} \subset \mathfrak{h}$ defined by

$$\mathscr{A} = \{\mu \in \mathfrak{h} : \alpha_i(\mu) \geq 0 \ \text{ and } \ \theta(\mu) \leq 1\},$$

where θ is the highest root of $\mathfrak{g}$. Then, $\mathscr{A}$ parameterizes the K-conjugacy classes of K under the map $C : \mathscr{A} \to K/\mathrm{Ad}\ K$,

$$\mu \mapsto c(Exp(2\pi i\mu)),$$

where K is a maximal compact subgroup of G and $c(Exp(2\pi i\mu))$ denotes the K-conjugacy class of $Exp(2\pi i\mu)$. Fix a positive integer $n \geq 2$ and define the *multiplicative polytope*

$$\mathscr{C}_n := \{(\mu_1, \ldots, \mu_n) \in \mathscr{A}^n : 1 \in C(\mu_1) \ldots C(\mu_n)\}.$$

Then, $\mathscr{C}_n$ is a rational convex polytope with nonempty interior in $\mathfrak{h}^n$. Our main result describes the facets (i.e., the codimension one faces) of $\mathscr{C}_n$. We construct deformations of the small quantum cohomology rings of homogeneous spaces G/P, and obtain an irredundant set of inequalities determining $\mathscr{C}_n$. The result was proved by Biswas in the case $G = SL_2$; by Belkale for $G = SL_m$ (and in this case a slightly weaker result by Agnihotri-Woodward); and by Teleman-Woodward for general G (though the result of Teleman-Woodward produced a set of inequalities which has redundancies in general).

Decomposition numbers and Lusztig induction

Gunter Malle
(TU Kaiserslautern)

There does not (yet) exist a general theory for describing decomposition numbers of finite reductive groups at primes different from the defining characteristic. We formulate a new conjecture predicting that characters of intersection cohomology complexes related to Lusztig induction should provide many new projective characters. We also present new, almost complete results on decomposition matrices of finite unitary groups of rank at most 10. This is joint work with Olivier Dudas.

Modular representation theory of hypertoric varieties

Carl Mautner
(MPIM Bonn)

(joint work with Tom Braden)

Let k be a field. We introduce a new family of highest weight categories (equivalently quasi-hereditary algebras) over k, which generalize the category of polynomial representations of GL_n (resp. the Schur algebra) over k.

Our definition originates from an observation that polynomial representations over k of GL_n has a geometric incarnation as a category of perverse sheaves with coefficients in k on the nilpotent cone of $GL_n(C)$. On the other hand, the nilpotent cone has a class of interesting relatives - affine hypertoric varieties. The latter are associated to hyperplane arrangements and have a strong combinatorial flavor.

For each affine hypertoric variety, we define a category of perverse sheaves with coefficients in k. We prove that the resulting category has a natural highest weight structure and that its Ringel dual is the category associated to the "Gale dual" hypertoric variety.

On good (p,r)-filtrations for rational G-modules

Daniel Nakano
(University of Georgia)

In this talk we investigate Donkin's (p, r)-Filtration Conjecture, and present two proofs of the "if" direction of the statement when $p \geq 2h - 2$. One proof involves the investigation of when the tensor product between the Steinberg module and a simple module has a good filtration. One of our main results shows that this holds under suitable requirements on the highest weight of the simple module. The second proof involves recasting Donkin's Conjecture in terms of the identifications of projective indecomposable G_r-modules with certain tilting G-modules, and establishing necessary cohomological criteria for the (p, r)-filtration conjecture to hold.

This is joint work with Tobias Kildetoft.

Covariants in the exterior algebra of a simple Lie algebra

Paolo Papi
(Sapienza Università di Roma)

For a simple complex Lie algebra $\mathfrak{g}$ we study the space of invariants $A = (\Lambda\mathfrak{g}^* \otimes \mathfrak{g}^*)^{\mathfrak{g}}$ (which describes the isotypic component of type $\mathfrak{g}$ in $\Lambda\mathfrak{g}^*$ as a module over the algebra of invariants $(\Lambda\mathfrak{g}^*)^{\mathfrak{g}}$. As main result we prove that A is a free module, of rank twice the rank of $\mathfrak{g}$, over the exterior algebra generated by all primitive invariants in $(\Lambda\mathfrak{g}^*)^{\mathfrak{g}}$, with the exception of the one of highest degree.

Joint work with C. De Concini and C. Procesi.

Bounding and unbounding cohomology for algebraic groups

Alison Parker
(University of Leeds)

Some new results/conjectures on the representation/cohomology theory of semisimple groups in positive characterisitc

Brian Parshall
(University of Virginia)

Recently, Leonard Scott and the speaker introduced the notion of a Q-Koszul algebra. I will discuss these algebras, and some related conjectures on the cohomology of semisimple groups. These conjectures allow the characteristic p to be very small (and there is no restriction at all in type A), and they give some specific calculations involving Kazhdan-Lusztig polynomials. I will mention some interesting examples supporting the conjectures.

Koszul duality for modular perverse sheaves on flag varieties

Simon Riche
(CNRS - Université Blaise Pascal)

I will explain how one can define a "Koszul duality" relating Bruhat-constructible sheaves on the flag variety of a reductive group, and similar sheaves on the flag variety of the Langlands dual group. In particular, this construction involves a new definition of the "mixed derived category" of constructible sheaves, which makes sense in the case of coefficients of positive characteristic. This talk will be based on a joint work with Pramod Achar.

D-Modules on Projective Stacks

Dmitriy Rumynin
(University of Warwick)

We will contemplate D-modules on weighted projective space and more general projective stacks. We will discuss examples of D-affine homogeneous projective stacks and whether a classification of such stacks is feasible.

Unicity of grading of category O

Wolfgang Soergel
(Universität Freiburg)

We explain how the usual grading on category $\mathcal{O}$ can be characterized by its compatibility with the action of the center of the enveloping algebra. This is joint work with Michael Rottmaier.

Smoothness of normalisers

David Stewart
(Cambridge)

One of the most basic questions one can ask about algebraic groups is if they are smooth. Centralisers of subgroups of reductive groups are reasonably well-behaved in this respect and are almost always smooth. By contrast normalisers are hardly ever smooth. I'll give some conditions on when they are in fact smooth and indicate implications for the subalgebra structure of modular Lie algebras. The proof uses results we prove about the vanishing of low degree cohomology of representations for Lie algebras in the style of Jantzen, Serre, Guralnick, Bendel-Nakano-Pillen.

Non-G-completely reducible subgroups in exceptional algebraic groups in good characteristic

Adam Thomas
(Imperial College London)

Let G be an exceptional algebraic group over an algebraically closed field of good characteristic. A subgroup X is said to be G-completely reducible if whenever it is contained in a parabolic subgroup P of G it is contained in a Levi subgroup of P. Work of Liebeck—Seitz and Stewart proves that all connected simple subgroups of G are completely reducible except for A_1 subgroups when $p = 5, 7$ and G_2 subgroups when $p = 7$. I will report on recent joint work with A. Litterick classifying such exceptions. This relies on both representation and cohomology theory of reductive groups, as well as computations in Magma.